Wave Propagation and Attenuation in the General Class of Circular Hollow Waveguides with Uniform Curvature

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Abstract —A general method has been developed to evaluate the propagation constant in oversized circular hollow-core waveguides characterized by a surface impedance and admittance due to a uniform bend. Completely different formulas are obtained for the attenuation constants of the modes in metallic or dielectric hollow waveguides from those obtained by Marcatili and Schmeltzer [1]. Electric-field lines are also presented for several lower order modes in bent waveguides.

I. Introduction

HOLLOW DIELECTRIC or metallic waveguides are possible transmission media when the absorptions of dielectric materials are too high to transmit the guided powers in dielectric media. When optical fibers with low losses were not available, hollow waveguides were proposed to carry optical signals, and very detailed analyses of them were done by Marcatili and Schmeltzer [1]. In connection with the delivery of infrared CO_2 laser energy, hollow waveguides are regarded again as transmission media, and several types of hollow waveguides have been proposed and fabricated [2]–[5]. One of the most serious problems in hollow waveguides is the increased loss due to bends.

To evaluate bending losses in circular waveguides, a theory presented by Marcatili and Schmeltzer [1] (hereafter referred to as the M-S theory) has been used over the past two decades in infrared as well as in submillimeter wavelengths [1], [6], [7]. However, as pointed out by Miyagi [8], the M-S theory didn't consider field deformations depending on R^{-2} (R; bending radius) which substantially affect the bending losses in their formulation, i.e., the power losses can be evaluated by the ratio of the radiated power from a waveguide per unit length and the total power carried by the mode of the waveguide. Recent experimental studies on bending losses of the TE_{01} mode in metallic waveguides at infrared [9] also suggest that the M-S theory should be modified so as to include high losses due to the mode coupling effect. A theory was developed by Marhic [10] to predict the bending losses by using the mode coupling analysis similar to one done for waveguides at the microwave region [11]. However, his analysis cannot be applied to infrared metallic waveguides where the imaginary part of the complex refractive index of the metals is much larger than its real part and the mode structure is different from that in waveguides at the microwave region [12]. Therefore, at present, we can say that there are no theories available to properly predict bending losses in circular dielectric or metallic waveguides at optical wavelengths through submillimeter wavelengths.

In this paper, we present a new method to evaluate the propagation constants in the general class of circular bent waveguides characterized by a surface impedance and admittance. The new method requires evaluation of the field deformations to R^{-1} only, not to R^{-2}. A general expression of the propagation constant is obtained which is valid to any circular waveguide, and explicit expressions of the phase and attenuation constants are given for waveguides with a small surface impedance and admittance. It is shown that the bending loss formulas obtained for the metallic or dielectric hollow waveguides are completely different from those of the M-S theory and more accurately explain the experimental results of high bending losses of the TE_{01} mode in circular metallic waveguides [9]. Electric-field lines of the modes in bent waveguides are also presented.

II. General Expressions of Field Distributions and Propagation Constant in Bent Waveguides

A. Integral Representation of the Change of Propagation Constant

Let a waveguide with a hollow-core radius T be bent uniformly with a large bending radius R. We employ the toroidal coordinate system [1] for analyses as shown in Fig. 1 and assume that the refractive index of the hollow-core is n_0 ($\simeq 1$), and media in $r > T$ are characterized by a normalized surface impedance z_{TE} and y_{TM} at $r = T$ [12], [13] as follows:

$$\left.\frac{E_\theta}{H_z}\right)_{r=T} = \frac{\omega\mu_0}{n_0 k_0} z_{TE}, \qquad \left.\frac{H_\theta}{E_z}\right)_{r=T} = -\frac{n_0 k_0}{\omega\mu_0} y_{TM}. \quad (1)$$

From the following Maxwell's equations in the hollow

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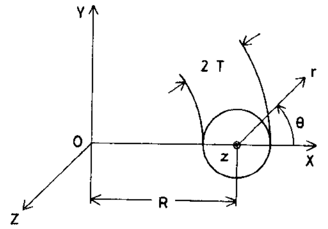


Fig. 1. Coordinate system for the curved circular hollow waveguide.

region:

$$\frac{1}{r}\frac{\partial}{\partial\theta}\left[\left(1+\frac{r}{R}\cos\theta\right)H_z\right]+j\beta H_\theta=j\omega\epsilon_0 n_0^2\left(1+\frac{r}{R}\cos\theta\right)E_r \quad (2)$$

$$-j\beta H_r-\frac{\partial}{\partial r}\left[\left(1+\frac{r}{R}\cos\theta\right)H_z\right]=j\omega\epsilon_0 n_0^2\left(1+\frac{r}{R}\cos\theta\right)E_\theta \quad (3)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH_\theta)-\frac{1}{r}\frac{\partial H_r}{\partial\theta}=j\omega\epsilon_0 n_0^2 E_z \quad (4)$$

$$\frac{1}{r}\frac{\partial}{\partial\theta}\left[\left(1+\frac{r}{R}\cos\theta\right)E_z\right]+j\beta E_\theta=-j\omega\mu_0\left(1+\frac{r}{R}\cos\theta\right)H_r \quad (5)$$

$$-j\beta E_r-\frac{\partial}{\partial r}\left[\left(1+\frac{r}{R}\cos\theta\right)E_z\right]=-j\omega\mu_0\left(1+\frac{r}{R}\cos\theta\right)H_\theta \quad (6)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta)-\frac{1}{r}\frac{\partial E_r}{\partial\theta}=-j\omega\mu_0 H_z \quad (7)$$

one can express the transverse field components by the axial components E_z and H_z as follows:

$$E_r=-j\frac{1}{n_0^2k_0^2\left(1+\frac{r}{R}\cos\theta\right)^2-\beta^2}\left[n_0k_0\frac{\partial E_z}{\partial r}+\frac{\omega\mu_0}{r}\frac{\partial H_z}{\partial\theta}\right] \quad (8)$$

$$E_\theta=-j\frac{1}{n_0^2k_0^2\left(1+\frac{r}{R}\cos\theta\right)^2-\beta^2}\left[\frac{n_0k_0}{r}\frac{\partial E_z}{\partial\theta}-\omega\mu_0\frac{\partial H_z}{\partial r}\right] \quad (9)$$

$$H_r=-\frac{n_0k_0}{\omega\mu_0}E_\theta,\quad H_\theta=\frac{n_0k_0}{\omega\mu_0}E_r \quad (10)$$

where time and z dependences of the form $\exp j(\omega t-\beta z)$ are suppressed, and it is assumed that

$$n_0k_0T\gg 1 \quad \text{and} \quad R\gg T \quad (11)$$

are needed to derive (8)–(10). One should also note that β is replaced by n_0k_0 inside the parentheses in (8) and (9).

By expanding $\mathbf{E}$, $\mathbf{H}$, and β as

$$\mathbf{E}=\mathbf{E}^{(0)}+\frac{1}{R}\mathbf{E}^{(1)}+\frac{1}{R^2}\mathbf{E}^{(2)}+\cdots \quad (12)$$

$$\mathbf{H}=\mathbf{H}^{(0)}+\frac{1}{R}\mathbf{H}^{(1)}+\frac{1}{R^2}\mathbf{H}^{(2)}+\cdots \quad (13)$$

$$\beta=\beta_0+\frac{1}{R^2}\delta\beta+\cdots \quad (14)$$

and substituting (12)–(14) into (8) and (9), one can express $E_r^{(i)}$ and $E_\theta^{(i)} (i=0,1,2)$ as follows:

$$E_r^{(i)}=\frac{2n_0k_0T^2}{u^2}\delta\beta E_r^{(i-2)}-2\left(\frac{n_0k_0T}{u}\right)^2 r\cdot\cos\theta E_r^{(i-1)}-j\left(\frac{T}{u}\right)^2\left[n_0k_0\frac{\partial E_z^{(i)}}{\partial r}+\frac{\omega\mu_0}{r}\frac{\partial H_z^{(i)}}{\partial\theta}\right] \quad (15)$$

$$E_\theta^{(i)}=\frac{2n_0k_0T^2}{u^2}\delta\beta E_\theta^{(i-2)}-2\left(\frac{n_0k_0T}{u}\right)^2 r\cdot\cos\theta E_\theta^{(i-1)}-j\left(\frac{T}{u}\right)^2\left[\frac{n_0k_0}{r}\frac{\partial E_z^{(i)}}{\partial\theta}-\omega\mu_0\frac{\partial H_z^{(i)}}{\partial r}\right] \quad (16)$$

where u is defined by

$$u^2=\left(n_0^2k_0^2-\beta_0^2\right)T^2. \quad (17)$$

Quantities with negative superscripts are understood to be zero. One should note that [1, eq. (34)], which describes the first-order perturbation solution, is incorrect as can be seen from (15) and (16) of the present paper.

By substituting the corresponding magnetic-field components $H_r^{(i)}$ and $H_\theta^{(i)}$ into (4), one obtains the differential equations to determine $E_z^{(i)} (i=0,1,2)$ as follows:

$$\nabla^2E_z^{(i)}+\left(\frac{u}{T}\right)^2E_z^{(i)}=2n_0k_0\delta\beta E_z^{(i-2)}-2n_0^2k_0^2r\cos\theta E_z^{(i-1)}+j2n_0k_0E_x^{(i-1)} \quad (18)$$

where

$$E_x^{(i-1)}=E_r^{(i-1)}\cos\theta-E_\theta^{(i-1)}\sin\theta \quad (19)$$

and ∇^2 is defined by

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}. \quad (20)$$

Similarly, one obtains

$$\nabla^2H_z^{(i)}+\left(\frac{u}{T}\right)^2H_z^{(i)}=2n_0k_0\delta\beta H_z^{(i-2)}-2n_0^2k_0^2r\cos\theta H_z^{(i-1)}+j2n_0k_0H_x^{(i-1)}. \quad (21)$$

Now, let us mention the evaluation method of $\delta\beta$. In the ordinary method of determining the change of the propagation constant due to bends, one has to derive a characteristic equation for $\delta\beta$ which requires solutions up to $\mathbf{E}^{(2)}$ and $\mathbf{H}^{(2)}$. To do this, extremely tedious and cumbersome calculations are needed. In this paper, we present a simple method which requires solutions only up to $\mathbf{E}^{(1)}$ and $\mathbf{H}^{(1)}$.

Making

$$E_z^{(0)}\nabla^2 E_z^{(2)} - E_z^{(2)}\nabla^2 E_z^{(0)} \tag{22}$$

by use of (18), and integrating in the hollow region, one obtains

$$\oint\left[E_z^{(0)}\frac{\partial E_z^{(2)}}{\partial r} - E_z^{(2)}\frac{\partial E_z^{(0)}}{\partial r}\right]dC = 2n_0k_0\int\left[\delta\beta E_z^{(0)^2} - n_0k_0 r\cos\theta E_z^{(0)}E_z^{(1)} + jE_x^{(1)}E_z^{(0)}\right]dS \tag{23}$$

where the Green's theorem was used to transform the surface integral in $r \leqslant T$ into the line integral around the periphery $r = T$ of the hollow region. Substituting $\partial E_z^{(0)}/\partial r$ and $\partial E_z^{(2)}/\partial r$ obtained from (15) into (23), and using the boundary conditions (1), one obtains

$$\begin{aligned}\delta\beta&\left[\int E_z^{(0)^2}dS + j\frac{1}{\omega\epsilon_0 n_0^2}\oint H_\theta^{(0)}E_z^{(0)}dC\right]\\ &= n_0k_0\int\left[r\cos\theta E_z^{(0)}E_z^{(1)} - j\frac{1}{\omega\epsilon_0 n_0^2}H_y^{(1)}E_z^{(0)}\right]dS\\ &\quad + j\frac{1}{\omega\epsilon_0 n_0^2}\oint\left\{n_0k_0 r\cos\theta E_z^{(0)}H_\theta^{(1)} - j\frac{1}{2r}\right.\\ &\quad\left.\cdot\left[E_z^{(2)}\frac{\partial H_z^{(0)}}{\partial\theta} - E_z^{(0)}\frac{\partial H_z^{(2)}}{\partial\theta}\right]\right\}dC\end{aligned} \tag{24}$$

where

$$H_y^{(1)} = H_r^{(1)}\sin\theta + H_\theta^{(1)}\cos\theta. \tag{25}$$

Similarly, integrating $H_z^{(0)}\nabla^2 H_z^{(2)} - H_z^{(2)}\nabla^2 H_z^{(0)}$ in the hollow region, one obtains

$$\begin{aligned}\delta\beta&\left[\int H_z^{(0)^2}dS - j\frac{1}{\omega\mu_0}\oint E_\theta^{(0)}H_z^{(0)}dC\right]\\ &= n_0k_0\int\left[r\cos\theta H_z^{(0)}H_z^{(1)} + j\frac{1}{\omega\mu_0}E_y^{(1)}H_z^{(0)}\right]dS\\ &\quad - j\frac{1}{\omega\mu_0}\oint\left\{n_0k_0 r\cos\theta H_z^{(0)}E_\theta^{(1)} + j\frac{1}{2r}\right.\\ &\quad\left.\cdot\left[H_z^{(0)}\frac{\partial E_z^{(2)}}{\partial\theta} - H_z^{(2)}\frac{\partial E_z^{(0)}}{\partial\theta}\right]\right\}dC.\end{aligned} \tag{26}$$

By making (24)$\times\,\omega\epsilon_0 n_0^2$ + (26)$\times\,\omega\mu_0$, one finally arrives at

$$\begin{aligned}\delta\beta&\left\{\int\left[\omega\epsilon_0 n_0^2 E_z^{(0)^2} + \omega\mu_0 H_z^{(0)^2}\right]dS\right.\\ &\quad\left.+ j\oint\left[H_\theta^{(0)}E_z^{(0)} - E_\theta^{(0)}H_z^{(0)}\right]dC\right\}\\ &= n_0k_0\left\{\int r\cos\theta\left[\omega\epsilon_0 n_0^2 E_z^{(0)}E_z^{(1)} + \omega\mu_0 H_z^{(0)}H_z^{(1)}\right]dS\right.\\ &\quad + j\int\left[E_y^{(1)}H_z^{(0)} - H_y^{(1)}E_z^{(0)}\right]dS\\ &\quad\left.+ j\oint r\cos\theta\left[H_\theta^{(1)}E_z^{(0)} - E_\theta^{(1)}H_z^{(0)}\right]dC\right\}.\end{aligned} \tag{27}$$

Equation (27) clearly shows that the change of the propagation constant can be evaluated by using only zeroth and first-order field distributions in a bent waveguide. The real and imaginary parts of $\delta\beta$ lead to the changes of the phase and attenuation constants of the modes due to bends, respectively.

B. Field Distributions and Complex Propagation Constant

In this section, we present the expressions of the field distributions and the propagation constant in the general circular waveguides with an arbitrary surface impedance and admittance.

As is well known, the zeroth order solutions $E_r^{(0)}$, $E_\theta^{(0)}$, $E_z^{(0)}$, and $H_z^{(0)}$ are given as follows [14]:

$$E_r^{(0)} = \left[\frac{1-P}{2}J_{n-1}(u\rho) - \frac{1+P}{2}J_{n+1}(u\rho)\right]\cos(n\theta+\theta_0) \tag{28}$$

$$E_\theta^{(0)} = -\left[\frac{1-P}{2}J_{n-1}(u\rho) + \frac{1+P}{2}J_{n+1}(u\rho)\right]\sin(n\theta+\theta_0) \tag{29}$$

$$E_z^{(0)} = j\frac{u}{n_0k_0T}J_n(u\rho)\cos(n\theta+\theta_0) \tag{30}$$

$$H_z^{(0)} = -j\left(\frac{n_0k_0}{\omega\mu_0}\right)\frac{u}{n_0k_0T}PJ_n(u\rho)\sin(n\theta+\theta_0) \tag{31}$$

where n is the integer describing the azimuthal dependence and ρ is defined by r/T. The normalized radial phase constant u and the parameter P deduced from the boundary conditions (1) are determined from

$$\left[\frac{J_n'(u)}{uJ_n(u)} + j\frac{z_{\rm TE}}{n_0k_0T}\right]\left[\frac{J_n'(u)}{uJ_n(u)} + j\frac{y_{\rm TM}}{n_0k_0T}\right] = \frac{n^2}{u^4} \tag{32}$$

$$P = \frac{u^2}{n}\left[\frac{J_n'(u)}{uJ_n(u)} + j\frac{y_{\rm TM}}{n_0k_0T}\right]. \tag{33}$$

By substituting (28)–(31) with (10) into (18) and (21), and integrating $E_z^{(1)}$ and $H_z^{(1)}$ (see Appendix A), one obtains

$$\begin{aligned}E_z^{(1)} = jn_0k_0T^2&\left\{\left[e_+J_{n+1}(u\rho) + \frac{1}{4}\rho^2 J_{n-1}(u\rho)\right.\right.\\ &\quad\left.+\frac{1+P}{2u}\rho J_n(u\rho)\right]\cos[(n+1)\theta+\theta_0]\\ &\quad+\left[e_-J_{n-1}(u\rho) - \frac{1}{4}\rho^2 J_{n+1}(u\rho)\right.\\ &\quad\left.\left.+\frac{1-P}{2u}\rho J_n(u\rho)\right]\cos[(n-1)\theta+\theta_0]\right\}\end{aligned} \tag{34}$$

$$\begin{aligned}H_z^{(1)} = j\left(\frac{n_0k_0}{\omega\mu_0}\right)n_0k_0T^2&\left\{\left[h_+J_{n+1}(u\rho) - \frac{P}{4}\rho^2 J_{n-1}(u\rho)\right.\right.\\ &\quad\left.-\frac{1+P}{2u}\rho J_n(u\rho)\right]\sin[(n+1)\theta+\theta_0]\\ &\quad+\left[h_-J_{n-1}(u\rho) + \frac{P}{4}\rho^2 J_{n+1}(u\rho)\right.\\ &\quad\left.\left.+\frac{1-P}{2u}\rho J_n(u\rho)\right]\sin[(n-1)\theta+\theta_0]\right\}.\end{aligned} \tag{35}$$

The transverse field components of $\mathbf{E}^{(1)}$ are calculated by using (15) and (16) as follows:

$$E_r^{(1)} = \frac{(n_0 k_0 T)^2 T}{u}\{\psi_{r_+}(\rho)\cos[(n+1)\theta+\theta_0] + \psi_{r_-}(\rho)\cos[(n-1)\theta+\theta_0]\} \quad (36)$$

$$E_\theta^{(1)} = -\frac{(n_0 k_0 T)^2 T}{u}\{\psi_{\theta_+}(\rho)\sin[(n+1)\theta+\theta_0] + \psi_{\theta_-}(\rho)\sin[(n-1)\theta+\theta_0]\} \quad (37)$$

where

$$\psi_{r_\pm}(\rho) = e_\pm J'_{n\pm1}(u\rho) + h_\pm \frac{n\pm1}{u\rho}J_{n\pm1}(u\rho) + \frac{(n\mp1)(1\mp P)}{4u}\rho J_{n\mp1}(u\rho) - \frac{1}{4}\rho^2 J_n(u\rho) \quad (38)$$

$$\psi_{\theta_\pm}(\rho) = h_\pm J'_{n\pm1}(u\rho) + e_\pm \frac{n\pm1}{u\rho}J_{n\pm1}(u\rho) \pm \frac{(n\mp1)(1\mp P)}{4u}\rho J_{n\mp1}(u\rho) + \frac{P}{4}\rho^2 J_n(u\rho). \quad (39)$$

The arbitrary constants $e_\pm$ and $h_\pm$ determined from the boundary conditions (1) are expressed as follows (see Appendix B):

$$e_\pm = \pm\frac{1}{4\Delta_{n\pm1}}\{[(n+P)J_{n\mp1} - uPZ_{n\mp1} \mp (1\pm P)(J_{n\pm1}\pm 2Z_n)](n\pm1)J_{n\pm1} + [(nP+1)J_{n\mp1} - uY_{n\mp1} - (1\pm P)(J_{n\pm1}\pm 2Y_n)]uZ_{n\pm1}\} \quad (40)$$

$$h_\pm = \mp\frac{1}{4\Delta_{n\pm1}}\{[(nP+1)J_{n\mp1} - uY_{n\mp1} - (1\pm P)(J_{n\pm1}\pm 2Y_n)](n\pm1)J_{n\pm1} + [(n+P)J_{n\mp1} - uPZ_{n\mp1} \mp (1\pm P)(J_{n\pm1}\pm 2Z_n)]uY_{n\pm1}\} \quad (41)$$

where the argument of the Bessel functions is u, and Z_n, Y_n, and Δ_n are defined by

$$Z_n = J_n'(u) + j\frac{u}{n_0 k_0 T} z_{\mathrm{TE}} J_n(u) \quad (42)$$

$$Y_n = J_n'(u) + j\frac{u}{n_0 k_0 T} y_{\mathrm{TM}} J_n(u) \quad (43)$$

$$\Delta_n = u^2 Z_n Y_n - n^2 J_n^2(u). \quad (44)$$

By substituting (34)–(37) into (27) and using some integral formulas containing the Bessel functions and powers of r (see Appendix C), we finally arrive at the general expression of $\delta\beta$ as follows:

$$\delta\beta = \frac{(n_0 k_0 T)^3 T}{2u^2}\Big\{(e_+ + e_-)[nJ_n^2 - (n+P)J_{n+1}J_{n-1}] - (h_+ + h_-)[nPJ_n^2 - (nP+1)J_{n+1}J_{n-1}] - (e_+ - h_+)(1+P)J_nJ_{n+2} + (e_- + h_-)\cdot(1-P)J_nJ_{n-2} + \frac{1}{12}[(1-P)^2(2J_nJ_{n-2} + J_{n+1}J_{n-3}) + (1+P)^2(2J_nJ_{n+2} + J_{n-1}J_{n+3})] + \delta_{n1}Q\cos 2\theta_0\Big\} \Big/ \Big\{(1+P^2)(J_n^2 - J_{n+1}J_{n-1}) + [(1-P)^2 J_{n-1} - (1+P)^2 J_{n+1}]\frac{J_n}{u}\Big\} \quad (45)$$

where

$$Q = (e_- P + h_-)J_1^2 - (e_- + h_-)\cdot(1+P)J_0J_2 + \frac{1}{4}(1-P^2)J_1J_3 \quad (46)$$

and the argument of Bessel functions is understood to be u. To evaluate e_- and h_- in (46), one has to put $n=1$ in (40) and (41). We have also proven that (45) is exactly the same with that obtained by using the characteristic equation after rather tedious calculations.

By dividing u into the real and imaginary parts as

$$u = u_0 + ju_t \quad (47)$$

one can deduce the changes of the phase and attenuation constants due to bends. However, it is rather cumbersome to derive them for arbitrary surface impedance and admittance. Furthermore, we are mainly interested in transmission of the HE_{11} mode in the infrared waveguides with a small surface impedance and admittance [12], [15]. Therefore, we will discuss this case in more detail in the next sections.

III. Wave Propagation and Attenuation in Circular Waveguides with Small Surface Impedance and Admittance

A. Phase and Attenuation Constants

When the following conditions are satisfied:

$$|z_{\mathrm{TE}}| \ll n_0 k_0 T/u_0, \quad |y_{\mathrm{TM}}| \ll n_0 k_0 T/u_0 \quad (48)$$

u_0 is determined from

$$J_1(u_0) = 0, \quad \mathrm{TE}_{0q} \text{ and } \mathrm{TM}_{0q} \text{ modes} \quad (49)$$

$$J_{n\mp1}(u_0) = 0, \quad \mathrm{HE}_{nq} \text{ and } \mathrm{EH}_{nq} \text{ modes} \quad (50)$$

and the parameter P becomes

$$P=\begin{cases}\infty, & \text{TE}_{0q}\text{ modes}\\ 0, & \text{TM}_{0q}\text{ modes}\\ \mp 1 - j\dfrac{u_0^2}{2n}\dfrac{z_{\text{TE}}-y_{\text{TM}}}{n_0k_0T}, & \text{HE}_{nq}\text{ and EH}_{nq}\text{ modes}\end{cases} \tag{51}$$

where the upper and lower lines in (50) and (51) correspond to the HE_{nq} and EH_{nq} modes, respectively, and only the first-order terms of z_{TE}/n_0k_0T and y_{TM}/n_0k_0T are considered in (51).

By substituting P and (49) into (40) and (41), and retaining the first-order terms of u_i, we can express $e_\pm$ and $h_\pm$ of the TE_{0q} modes as

$$e_+ = e_- = j\frac{1}{2u_0}\left(u_i - \frac{u_0}{n_0k_0T}y_{\text{TM}}\right)P \tag{52}$$

$$h_+ = -h_- = -\frac{1}{4}\left(1 - j\frac{2u_i}{u_0}\right)P \tag{53}$$

and $\delta\beta$ as follows:

$$\delta\beta = \frac{(n_0k_0T)^3T}{12u_0^2}\left[1 + j\frac{2}{u_0}\left(u_i - 3\frac{u_0}{n_0k_0T}y_{\text{TM}}\right)\right] \tag{54}$$

which leads to

$$\text{Re}(\beta) = n_0k_0\left[1 + \frac{1}{12}\left(\frac{n_0k_0T}{u_0}\right)^2\left(\frac{T}{R}\right)^2\right]. \tag{55}$$

As u_i is related to the attenuation constant α_∞ in the straight waveguide as

$$u_iu_0 = n_0k_0T^2\alpha_\infty \tag{56}$$

and α_∞ is

$$\alpha_\infty = n_0k_0\frac{u_0^2}{(n_0k_0T)^3}\text{Re}(z_{\text{TE}}) \tag{57}$$

for the TE_{0q} modes, one can express the attenuation constants α of the TE_{0q} modes in the bent waveguides as follows:

$$\alpha = \alpha_\infty\left\{1 - \frac{1}{6}\left(\frac{n_0k_0T}{u_0}\right)^4\left(\frac{T}{R}\right)^2\left[1 - 3\frac{\text{Re}(y_{\text{TM}})}{\text{Re}(z_{\text{TE}})}\right]\right\}. \tag{58}$$

For the TM_{0q} modes, one has

$$e_+ = -e_- = \frac{1}{4}\left(1 - j\frac{2u_i}{u_0}\right) \tag{59}$$

$$h_+ = h_- = -j\frac{1}{2u_0}\left(u_i - \frac{u_0}{n_0k_0T}z_{\text{TE}}\right) \tag{60}$$

and

$$\delta\beta = \frac{(n_0k_0T)^3T}{12u_0^2}\left[1 + j\frac{2}{u_0}\left(u_i - 3\frac{u_0}{n_0k_0T}z_{\text{TE}}\right)\right] \tag{61}$$

TABLE I
PHASE AND ATTENUATION CONSTANTS OF THE MODES IN CURVED CIRCULAR HOLLOW WAVEGUIDES WITH A NORMALIZED SURFACE IMPEDANCE z_{TE} AND ADMITTANCE y_{TM}.

MODE	PHASE CONSTANT $\text{Re}(\beta)/n_0k_0$	ATTENUATION CONSTANT α/α_∞	u_0
TE_{0q}	$1+\frac{1}{12}\left(\frac{n_0k_0T}{u_0}\right)^2\left(\frac{T}{R}\right)^2$	$1-\frac{1}{6}\left(\frac{n_0k_0T}{u_0}\right)^4\left(\frac{T}{R}\right)^2\left[1-3\frac{\text{Re}(y_{\text{TM}})}{\text{Re}(z_{\text{TE}})}\right]$	$J_1(u_0)=0$
TM_{0q}		$1-\frac{1}{6}\left(\frac{n_0k_0T}{u_0}\right)^4\left(\frac{T}{R}\right)^2\left[1-3\frac{\text{Re}(z_{\text{TE}})}{\text{Re}(y_{\text{TM}})}\right]$	
HE_{nq} EH_{nq}	$1+\frac{1}{12}\left(\frac{n_0k_0T}{u_0}\right)^2\left(\frac{T}{R}\right)^2\left[1-\frac{4n(n\mp2)}{u_0^2}\right]$	$1+\frac{1}{3}\left(\frac{n_0k_0T}{u_0}\right)^4\left(\frac{T}{R}\right)^2\left[1-\frac{4n(n\mp2)}{u_0^2} \pm\frac{3}{8}\delta_{n1}\frac{\text{Re}(y_{\text{TM}}-z_{\text{TE}})}{\text{Re}(y_{\text{TM}}+z_{\text{TE}})}(u_0^2-2)\cos2\theta_0\right]$	$J_{n\mp1}(u_0)=0$

which leads to the attenuation constants as

$$\alpha = \alpha_\infty\left\{1 - \frac{1}{6}\left(\frac{n_0k_0T}{u_0}\right)^4\left(\frac{T}{R}\right)^2\left[1 - 3\frac{\text{Re}(z_{\text{TE}})}{\text{Re}(y_{\text{TM}})}\right]\right\} \tag{62}$$

where

$$\alpha_\infty = n_0k_0\frac{u_0^2}{(n_0k_0T)^3}\text{Re}(y_{\text{TM}}). \tag{63}$$

The expressions (58) and (62) clearly show the coupling effect between the TE_{0q} and TM_{0q} modes.

For the hybrid modes, i.e., $n \neq 0$, by using $e_\pm$ and $h_\pm$ in Appendix D, we can express $\delta\beta$ as follows:

$$\begin{aligned}\delta\beta = \frac{(n_0k_0T)^3T}{12u_0^2}\Bigg\{1 &- \frac{4n(n\mp2)}{u_0^2}\\ &+ j\frac{8u_i}{u_0}\left[1 + \frac{3}{4}n(n\mp2) + \frac{2n(n\mp2)}{u_0^2}\right]\\ &- j\frac{3(z_{\text{TE}}+y_{\text{TM}})}{2n_0k_0T}\left[n^2 + (n\mp2)^2\right]\\ &\pm j\delta_{n1}\frac{3(z_{\text{TE}}-y_{\text{TM}})}{4n_0k_0T}(u_0^2-2)\cos2\theta_0\Bigg\}.\end{aligned} \tag{64}$$

Table I summarizes the phase and attenuation constants of all modes normalized by n_0k_0 and α_∞ in the straight waveguide, respectively, where the attenuation constants α_∞ of the hybrid modes are

$$\alpha_\infty = \frac{1}{2}n_0k_0\frac{u_0^2}{(n_0k_0T)^3}\text{Re}(z_{\text{TE}} + y_{\text{TM}}). \tag{65}$$

It should be noted that only the attenuation constants of the HE_{1q} and EH_{1q} modes depend on the orientation θ_0 [1].

Let us compare the above results of the attenuation constants in the dielectric or metallic hollow waveguides with those of the M-S theory. Let a complex refractive index of the medium outside the core be $n_0\nu$. By using a simple method to calculate the normalized surface imped-

TABLE II
NORMALIZED ATTENUATION CONSTANTS α/α_∞ IN CURVED DIELECTRIC OR METALLIC HOLLOW WAVEGUIDES PREDICTED BY THE M-S THEORY AND OURS

MODE	M-S THEORY	PRESENT THEORY
TE_{0q}	$1+\frac{4}{3}\left(\frac{n_0k_0T}{u_0}\right)^4\left(\frac{T}{R}\right)^2$	$1-\frac{1}{6}\left(\frac{n_0k_0T}{u_0}\right)^4\left(\frac{T}{R}\right)^2\left\{1-3\frac{\mathrm{Re}\left[\frac{\nu^2}{(\nu^2-1)^{1/2}}\right]}{\mathrm{Re}\left[\frac{1}{(\nu^2-1)^{1/2}}\right]}\right\}$
TM_{0q}		$1-\frac{1}{6}\left(\frac{n_0k_0T}{u_0}\right)^4\left(\frac{T}{R}\right)^2\left\{1-3\frac{\mathrm{Re}\left[\frac{1}{(\nu^2-1)^{1/2}}\right]}{\mathrm{Re}\left[\frac{\nu^2}{(\nu^2-1)^{1/2}}\right]}\right\}$
HE_{nq} EH_{nq}	$1+\frac{4}{3}\left(\frac{n_0k_0T}{u_0}\right)^4\left(\frac{T}{R}\right)^2\left\{1-\frac{n(n\mp 2)}{u_0^2}+\frac{3}{4}\delta_{n1}\frac{\mathrm{Re}\left[(\nu^2-1)^{1/2}\right]}{\mathrm{Re}\left[\frac{\nu^2+1}{(\nu^2-1)^{1/2}}\right]}\cos 2\theta_0\right\}$	$1+\frac{1}{3}\left(\frac{n_0k_0T}{u_0}\right)^4\left(\frac{T}{R}\right)^2\left\{1-\frac{4n(n\mp 2)}{u_0^2}\pm\frac{3}{8}\delta_{n1}\frac{\mathrm{Re}\left[(\nu^2-1)^{1/2}\right]}{\mathrm{Re}\left[\frac{\nu^2+1}{(\nu^2-1)^{1/2}}\right]}(u_0^2-2)\cos 2\theta_0\right\}$

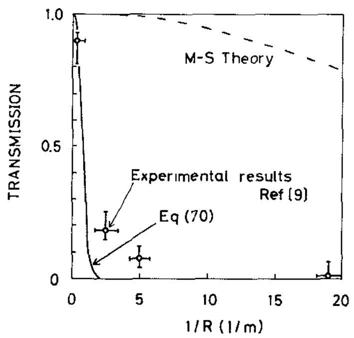


Fig. 2. Transmission versus curvature for the TE_{01} mode in a circular silver waveguide with $n = 13.4$, $\kappa = 75.2$, $2T = 500$ μm, and length of 26 cm at $\lambda = 10.6$ μm. Dashed and solid lines represent the transmission predicted by the M-S theory and ours, respectively.

ance and admittance [15], one obtains

$$z_{TE} = (\nu^2-1)^{-1/2} \tag{66}$$

$$y_{TM} = \nu^2(\nu^2-1)^{-1/2}. \tag{67}$$

Therefore, the attenuation constants of the modes can be summarized as in Table II. The difference between the M-S and our theories is clearly shown.

For the metallic hollow waveguides with $\nu = n - j\kappa$, where n and κ are much greater than unity at the infrared, we have

$$\mathrm{Re}(z_{TE}) = \frac{n}{n^2+\kappa^2} \tag{68}$$

$$\mathrm{Re}(y_{TM}) = n \tag{69}$$

and

$$\alpha/\alpha_\infty = 1 + \frac{1}{2}\left(\frac{n_0k_0T}{u_0}\right)^4\left(\frac{T}{R}\right)^2(n^2+\kappa^2) \tag{70}$$

for the TE_{0q} modes. By considering the fact that the values of $n^2+\kappa^2$ in most metals at the infrared are about several thousands, one can see that (70) is quite different from that of the M-S theory. In fact, (70) can more accurately explain the measured high curvature losses of the TE_{01} mode in metallic waveguides as shown in Fig. 2. Our theory also predicts that the attenuation constants of the TM_{0q} and several hybrid modes reduce due to gradual bends as shown in Figs. 3 and 4, which is very similar to the

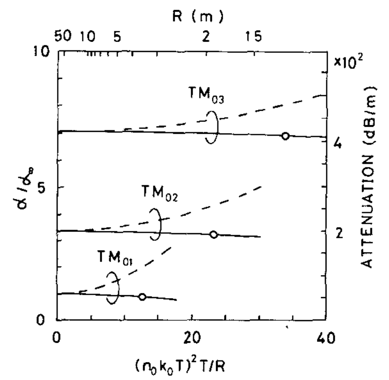


Fig. 3. Attenuation constants of the TM_{0q} modes in a bent aluminum waveguide with $n = 20.5$ and $\kappa = 58.6$. The dashed and solid lines are α/α_∞ predicted by the M-S theory and ours, respectively. Right and upper scales in the figure correspond to the attenuation and the bending radius of the waveguide with $T = 500$ μm at $\lambda = 10.6$ μm. Small circles correspond to the upper limit of applicability of the theories (see Appendix E).

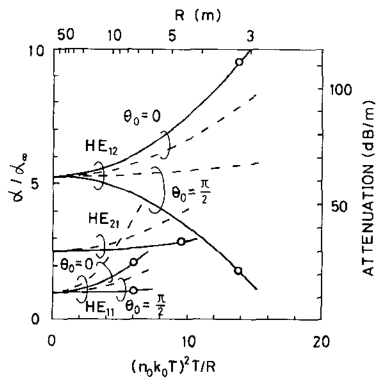


Fig. 4. Attenuation constants of the HE_{nq} modes in a bent aluminum waveguide. Parameters are the same with those in Fig. 3.

attenuation constants of higher order modes in slab waveguides [8], [16].

B. Electric-Field Lines of Modes in Bent Waveguides

In straight circular hollow waveguides where $|z_{TE}|/n_0k_0T$ and $|y_{TM}|/n_0k_0T$ are small, we have sufficient knowledge about the electric- and magnetic-field lines of the modes [1]. However, no descriptions of them have ever appeared in bent waveguides.

By simplifying the coefficients $e_\pm$ and $h_\pm$, and the parameter P by neglecting small quantities of u_t, z_{TE}/n_0k_0T, and y_{TM}/n_0k_0T, we can express the transverse-field components of the hybrid modes as follows:

$$E_r = J_{n\mp1}(u_0\rho)\cos(n\theta+\theta_0) + \frac{(n_0k_0T)^2T}{4u_0R} \cdot \{g_{n\mp1}(\rho)\cos[(n+1)\theta+\theta_0] - h_{n\mp1}(\rho)\cos[(n-1)\theta+\theta_0]\} \tag{71}$$

$$E_\theta = \mp J_{n\mp1}(u_0\rho)\sin(n\theta+\theta_0) \mp \frac{(n_0k_0T)^2T}{4u_0R} \cdot \{g_{n\mp1}(\rho)\sin[(n+1)\theta+\theta_0] - h_{n\mp1}(\rho)\sin[(n-1)\theta+\theta_0]\} \tag{72}$$

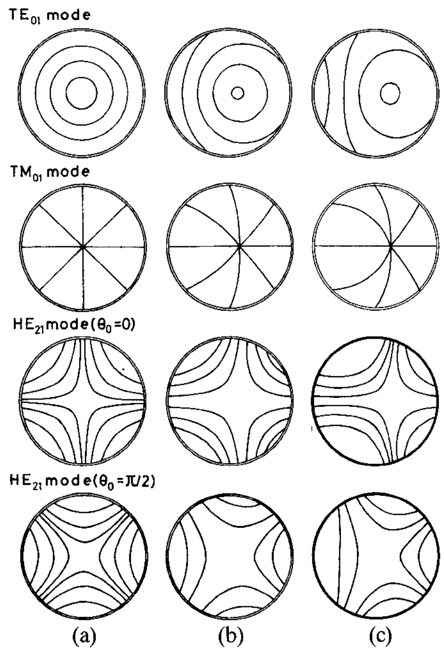


Fig. 5. Electric-field lines of the modes in curved hollow waveguides for (a) $(n_0k_0T)^2T/R=0$, (b) 5, and (c) 10, respectively.

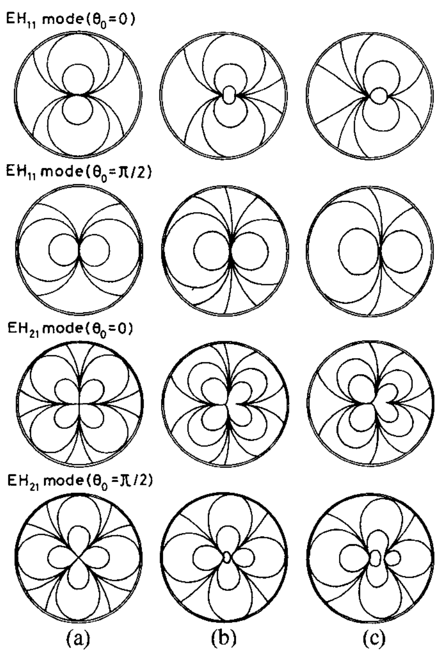


Fig. 6. Electric-field lines of the modes in curved hollow waveguides for (a) $(n_0k_0T)^2T/R=0$, (b) 5, and (c) 10, respectively.

where

$$g_n(\rho)=J_{n+1}(u_0\rho)+\rho^2J_{n-1}(u_0\rho) \tag{73}$$

$$h_n(\rho)=J_{n-1}(u_0\rho)+\rho^2J_{n+1}(u_0\rho) \tag{74}$$

and the upper and lower lines correspond to the HE_{nq} and EH_{nq} modes, respectively. The transverse-field components of the $\mathrm{TE}_{0q}(\mathrm{TM}_{0q})$ modes are simply obtained by putting $n=0$ and $\theta_0=\pi/2(0)$ in the expression of the $\mathrm{HE}_{nq}(\mathrm{EH}_{nq})$ modes.

Electric-field lines of the modes are determined from

$$\frac{1}{r}\frac{dr}{d\theta}=\frac{E_r}{E_\theta} \tag{75}$$

and magnetic-field lines are perpendicular to the electric-field lines in $r<T$ as seen in (10). One should note that the electric-field lines of the HE_{1q} modes never change due to bends [17], although the field distributions change.

Several numerical results obtained by integrating (75) by the use of the Runge-Kutta method are shown in Figs. 5 and 6 for several lower order modes. Generally speaking, electric-field lines become unsymmetrical, and some characteristic features peculiar to the modes, e.g., the center of the electric-field lines in the TE_{01} mode and a crossing point in the TM_{01} mode shift toward the outer direction away from the center of the curvature. Although field lines behave in complicated ways near the plane of the curvature, they are not so essential because the intensities of the modes are weak there.

IV. Conclusion

A general method is presented to evaluate the propagation constant in circular hollow waveguides. Completely different formulas are obtained for the attenuation constants of the modes in metallic or dielectric waveguides from those obtained by Marcatili and Schmeltzer [1].

Attenuation characteristics of the dielectric-coated metallic waveguide [12], [15] will be discussed elsewhere based on the new bending loss formulas.

Appendix A

Differential equations for $E_z^{(1)}$ and $H_z^{(1)}$ are described as

$$\begin{aligned}\nabla^2E_z^{(1)}+\left(\frac{u}{T}\right)^2E_z^{(1)}\\ =-jn_0k_0\{&[u\rho J_n(u\rho)+(1+P)J_{n+1}(u\rho)]\\ &\cdot\cos[(n+1)\theta+\theta_0]\\ &+[u\rho J_n(u\rho)-(1-P)J_{n-1}(u\rho)]\\ &\cdot\cos[(n-1)\theta+\theta_0]\}\end{aligned} \tag{A1}$$

$$\begin{aligned}\nabla^2H_z^{(1)}+\left(\frac{u}{T}\right)^2H_z^{(1)}\\ =j\left(\frac{n_0k_0}{\omega\mu_0}\right)n_0k_0\{&[Pu\rho J_n(u\rho)+(1+P)J_{n+1}(u\rho)]\\ &\cdot\sin[(n+1)\theta+\theta_0]\\ &+[Pu\rho J_n(u\rho)+(1-P)J_{n-1}(u\rho)]\\ &\cdot\sin[(n-1)\theta+\theta_0]\}.\end{aligned} \tag{A2}$$

Appendix B

From the boundary conditions (1), we obtain the equations to determine $e_\pm$ and $h_\pm$ as follows:

$$\psi_{r_\pm}(1)=-j\frac{u}{n_0k_0T}y_{\mathrm{TM}}\cdot\left[e_\pm J_{n\pm1}(u)\pm\frac{1}{4}J_{n\mp1}(u)+\frac{1\pm P}{2u}J_n(u)\right] \tag{B1}$$

$$\psi_{\theta_\pm}(1) = -j\frac{u}{n_0k_0T}z_{\mathrm{TE}} \cdot\left[h_\pm J_{n\pm1}(u) \mp \frac{P}{4}J_{n\mp1}(u) \mp \frac{1\pm P}{2u}J_n(u)\right]. \tag{B2}$$

Appendix C

Several integral formulas to evaluate $\delta\beta$ of (45) are listed as follows:

$$\int J_n^2(x)x\,dx = \frac{x^2}{2}\left(J_n^2 - J_{n+1}J_{n-1}\right) \tag{C1}$$

$$\int J_{n\pm1}(x)J_n(x)x^2\,dx = \frac{x^3}{4}\left(J_{n\pm1}J_n - J_{n\pm2}J_{n\mp1}\right) \tag{C2}$$

$$\int J_{n\pm2}(x)J_n(x)x^3\,dx = \frac{x^4}{6}\left(J_{n\pm2}J_n - J_{n\pm3}J_{n\mp1}\right) \tag{C3}$$

$$\int J_n^2(x)x^3\,dx = \frac{x^4}{12}\left(3J_n^2 - 2J_{n+1}J_{n-1} - J_{n+2}J_{n-2}\right) \tag{C4}$$

$$\int J_{n\pm1}(x)J_n(x)x^4\,dx = \frac{x^5}{24} \cdot\left(4J_{n\pm1}J_n - 3J_{n\pm2}J_{n\mp1} - J_{n\mp2}J_{n\pm3}\right) \tag{C5}$$

where the argument of the Bessel functions in the right-hand sides of (C1)–(C5) is x.

Appendix D

For the HE_{nq} modes, $e_\pm$ and $h_\pm$ are expressed as

$$e_+ = \frac{1}{4} - j\frac{1}{2u_0}\left[(n+1)\left(1-4\frac{n}{u_0^2}\right)u_t - n \cdot\frac{u_0}{n_0k_0T}z_{\mathrm{TE}} - \frac{n(n+1)}{u_0}P_t\right]\left[1-\frac{4n(n+1)}{u_0^2}\right]^{-1} \tag{D1}$$

$$e_- = -\frac{1}{4} + j\frac{1}{2u_0}\left[(n-1)u_t - (n-2) \cdot\frac{u_0}{n_0k_0T}y_{\mathrm{TM}} + \frac{n(n+1)}{u_0}P_t\right] \tag{D2}$$

$$h_+ = \frac{1}{4} - j\frac{1}{2u_0}\left\{(n+1)\left(1-\frac{4n}{u_0^2}\right)u_t - n\frac{u_0}{n_0k_0T}y_{\mathrm{TM}} + \left[u_0 - \frac{2n(n+1)}{u_0}\right]\frac{P_t}{2}\right\}\left[1-\frac{4n(n+1)}{u_0^2}\right]^{-1} \tag{D3}$$

$$h_- = -\frac{1}{4} + j\frac{1}{2u_0}\left\{(n-1)u_t - (n-2) \cdot\frac{u_0}{n_0k_0T}z_{\mathrm{TE}} + \left[u_0 - \frac{2n(n+1)}{u_0}\right]\frac{P_t}{2}\right\} \tag{D4}$$

where

$$P_t = \frac{u_0^2}{2n}\,\frac{y_{\mathrm{TM}} - z_{\mathrm{TE}}}{n_0k_0T}. \tag{D5}$$

To obtain the coefficients $e_\pm$ and $h_\pm$ for the EH_{nq} modes, we first change n to $-n$ in (D1)–(D5) and only replace $-e_-$, $-e_+$, h_-, and h_+ of the HE_{nq} modes by e_+, e_-, h_+, and h_- of the EH_{nq} modes.

Appendix E

We discuss the range of applicability of bending loss formulas given in Table I. For the electric and magnetic fields obtained by the perturbation theory to describe the actual fields properly, it is necessary that

$$R \gg R_l \tag{E1}$$

where R_l is defined by

$$|E_z^{(0)}|_{\max} = \frac{1}{R_l}|E_z^{(1)}|_{\max} \quad \text{or} \quad |H_z^{(0)}|_{\max} = \frac{1}{R_l}|H_z^{(1)}|_{\max}. \tag{E2}$$

For the TE_{0q} and TM_{0q} modes, we obtain

$$R_l = \frac{(n_0k_0T)^2T}{2u_0}\left|(1-\rho^2)J_1(u_0\rho) + \frac{2}{u_0}\rho J_0(u_0\rho)\right|_{\max} \tag{E3}$$

and

$$R_l = \frac{(n_0k_0T)^2T}{4u_0}\Big[|J_{n\mp1}(u_0\rho)| + \left|\rho^2J_{n\pm1}(u_0\rho) \mp \frac{4}{u_0}\rho J_n(u_0\rho)\right| + |J_{n\pm1}(u_0\rho) + \rho^2J_{n\mp1}(u_0\rho)|\Big]_{\max} \Big/ |J_n(u_0\rho)|_{\max} \tag{E4}$$

for the hybrid modes, where the upper and lower lines correspond to the HE_{nq} and EH_{nq} modes, respectively. Equations (E3) and (E4) are obtained by using the simplified coefficients $e_\pm$ and $h_\pm$, where the small quantities of u_t, z_{TE}/n_0k_0T, and y_{TM}/n_0k_0T are neglected.

By putting

$$R_l = A\frac{(n_0k_0T)^2T}{u_0} \tag{E5}$$

and calculating the parameter A numerically, we find that

$$A \simeq \begin{cases} 0.3, & \mathrm{TE}_{0q} \text{ and } \mathrm{TM}_{0q} \text{ modes} \\ 0.4, & \mathrm{HE}_{nq} \text{ modes} \\ 0.6, & \mathrm{EH}_{nq} \text{ modes} \end{cases} \tag{E6}$$

for several lower order modes of $n = 1, 2, 3$ and $q = 1, 2,\cdots,5$.

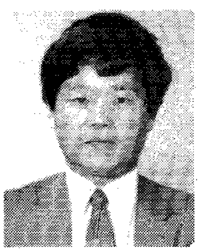
References

[1] E. A. J. Marcatili and R. A. Schmeltzer, "Hollow metallic and dielectric waveguides for long distance optical transmission and lasers," *Bell Syst. Tech. J.*, vol. 43, pp. 1783–1809, July 1964.

[2] E. Garmire, T. McMahon, and M. Bass, "Flexible infrared waveguides for high-power transmission," *IEEE J. Quantum Electron.*, vol. QE-16, pp. 23–32, Jan. 1980.

[3] M. E. Marhic, L. I. Kwan, and M. Epstein, "Optical surface waves along a toroidal metallic guide," *Appl. Phys. Lett.*, vol. 33, pp. 609–611, Oct. 1978.

[4] T. Hidaka, T. Morikawa, and J. Shimada, "Oxide-glass cladding middle infrared optical waveguides," *Trans. Inst. Electron. Commun. Eng. Japan*, vol. J64-C, pp. 590–596, Sept. 1981 (in Japanese).
[5] M. Miyagi, Y. Aizawa, A. Hongo, and S. Kawakami, "Fabricating dielectric-coated metallic hollow waveguides for IR transmission: A novel technique," in *Tech. Dig. CLEO'83*, 1983, pp. 210–211. See also M. Miyagi, A. Hongo, Y. Aizawa, and S. Kawakami, "Fabrication of germanium-coated nickel hollow waveguides for infrared transmission," *Appl. Phys. Lett.*, vol. 43, pp. 430–432, Sept. 1983.
[6] E. Garmire, T. McMahon, and M. Bass, "Propagation of infrared light in flexible hollow waveguides," *Appl. Opt.*, vol. 15, pp. 145–150, Jan 1976.
[7] F. K. Kneubühl and E. Affolter, "Infrared and submillimeter-wave waveguides," *Infrared and Millimeter Waves, vol. 1, Sources of Radiation*, K. J. Button, ed. New York: Academic Press, 1979, pp. 235–278.
[8] M. Miyagi, "Bending losses in hollow and dielectric tube leaky waveguides," *Appl. Opt.*, vol. 20, pp. 1221–1229, Apr. 1981.
[9] M. E. Marhic and E. Garmire, "Low-order TE_{0q} operation of a CO_2 laser for transmission through circular metallic waveguides," *Appl. Phys. Lett.*, vol. 38, pp. 743–745, May 1981.
[10] M. E. Marhic, "Mode-coupling analysis of bending losses in IR metallic waveguides," *Appl. Opt.*, vol. 20, pp. 3436–3441, Oct. 1981.
[11] H. G. Unger, "Lined waveguide," *Bell Syst. Tech. J.*, vol. 41, pp. 745–768, Mar. 1962.
[12] M. Miyagi and S. Kawakami, "Design theory of dielectric-coated circular metallic hollow waveguides for infrared transmission," *IEEE J. Lightwave Technol.*, vol. LT-2, no. 2, pp. 000–000, Apr. 1984.
[13] A. E. Karbowiak, "Theory of imperfect waveguides: The effect of wall impedance," *Proc. Inst. Elec. Eng.*, vol. 102, pp. 698–708, Sept. 1955.
[14] E. Snitzer, "Cylindrical dielectric waveguide modes," *J. Opt. Soc. Am.*, vol. 51, pp. 491–498, May 1961.
[15] M. Miyagi, A. Hongo, and S. Kawakami, "Transmission characteristics of dielectric-coated metallic waveguide for infrared transmission: Slab waveguide model," *IEEE J. Quantum Electron.*, vol. QE-19, pp. 136–145, Feb. 1983.
[16] H. Krammer, "Propagation of modes in curved hollow waveguides for the infrared," *Appl. Opt.*, vol. 16, pp. 2163–2165, Aug. 1977.
[17] M. Miyagi and G. L. Yip, "Field deformation and polarization change in a step-index optical fibre due to bending," *Opt. Quantum Electron.*, vol. 8, pp. 335–341, 1976.



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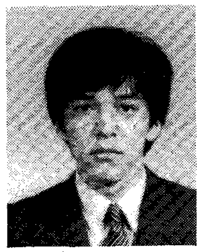
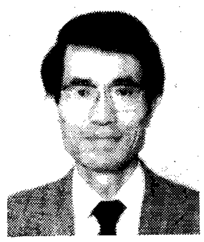
Dr. Miyagi is a member of the Institute of Electronics and Communication Engineers of Japan, Optical Society of America, and American Institute of Physics.



Kazuhide Harada was born in Aichi, Japan, on January 23, 1960. He received the B.E. degree from Gifu University, Gifu, Japan, in 1960. He is working towards the M.E. degree at Tohoku University, where he is studying IR waveguides.

Mr. Harada is a member of the Institute of Electronics and Communication engineers of Japan.



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